**Math Challenge #1**

In mathematics, there are many things that we *believe* are true but no one has been able to *prove*, or *explain why*, they are true. Such problems are called *open problems* because they are "open" for someone to come and solve them. For example, **Fermat's Last Theorem** states that:

"No three positive integers satisfy the equation $a^n + b^n = c^n$ when n is an integer greater than 2."

For 358 years, no one was able to prove this simple fact. It was only in 1994 that a British mathematician named Andrew Wiles proved this to be true.

Similarly, **the Collatz conjecture** states the following:

"Take any positive integer *n*. If *n* is even, divide it by 2. If *n* is odd, multiply it by 3 and add 1. Repeat the process over and over. Eventually, you will reach the number 1."

In mathematical notation, we can write this as a *function*:

$$f(n) = \begin{cases} n/2 & \text{if } n \equiv 0 \pmod 2 \\ 3n+1 & \text{if } n \equiv 1 \pmod 2. \end{cases}$$

As an example, let's use 13 as our number for *n*. 13 is odd, so we first multiply it by 3 and add 1 to get 40. The number 40 is even, so we divide it by 2 to get 20. The number 20 is even, so we divide it by 2 to get 10. After awhile, we get 1:

$$13 \rightarrow 40 \rightarrow 20 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$$

Almost everyone agrees that no matter what we choose for *n*, we will always get 1. But unlike Fermat's Last Theorem, ***no one is able to explain why this is true***. One mathematician says, "this is an extraordinarily difficult problem, completely out of reach of present day mathematics."

If you are able to prove the Collatz conjecture, you can quit school now to become rich and famous. But for now, can you find the number between 2 and 1,000,000 that creates the longest Collatz sequence? In our example above, you can count for yourself and see that the number 13 has **10 terms** in its Collatz sequence, but other numbers have longer and shorter sequences. Find the number that has the longest one.

Clearly, this is a problem you need a computer to help you solve. You know how to use *for* loops, *while* loops, *if* statements, and variables in Python – how can you use those to write a program that solves this problem for you? Your program should print out the number with the longest Collatz sequence **and** the length of that sequence.

**A final note of advice:** even for a computer, generating one million Collatz sequences is a lot of work. When writing computer programs that work with large numbers like this, it's often easier to use small numbers first, then use big numbers only when your program is ready. So for this problem, first make your Python program find the number between 2 and **100** that creates the longest Collatz sequence. The answer is 97, and it has a Collatz sequence of length 119. If your program can find that answer successfully, you'll be ready to replace 100 with 1,000,000 to find the answer to this problem.